Single-Field Radial Point Interpolation Method (RPIM) for Long-Range Propagation Modeling

Kazem Sabet and Anca I. Stefan
EMAG Technologies Inc.
Ann Arbor, MI, USA
ksabet@emagtech.com, astefan@emagtech.com

***Abstract*—The computational performance of the radial point interpolation method (RPIM) based on the second order wave equation is analyzed in a two-dimensional wave propagation scenario. Our results indicate that RPIM may outperform FDTD where simulations of electrically large problems are concerned and maintenance of phase integrity is desired.**

Keywords—finite difference methods; interpolation

I. Introduction

The radial point interpolation method (RPIM) has elicited interest from the computational electromagnetics community in the past decade [1]-[4]. A major advantage of RPIM over the finite-difference time domain (FDTD) method lies in its ability to solve problems on complicated geometries that would otherwise lead to very refined FDTD grids, require long computation times, or initiate signal integrity issues when modeling wave propagation over large distances. RPIM uses a weighted sum of a set of shifted replicates of a given basis function. These replicates have a known expression and are differentiated analytically. Therefore, RPIM can be used to reduce the FDTD dispersion effects, another potential advantage over FDTD. Commonly-used basis functions in RPIM are functions that depend on one parameter, termed radial basis functions (RBFs).

RPIM can overcome some of the known issues of pseudo-spectral time domain (PSTD) methods [5]. For example, the presence of the Gibbs phenomenon at the boundaries, the requirement of uniform or Chebyshev-type grids, as well as computational challenges presented by large-scale problems, are all characteristic of PSTD with global or local Fourier bases. However, the use of RPIM in time marching schemes entails a number of issues, such as late-time instability, which becomes critical when modeling electrically large problems [2]-[3].

In the work presented here, we discuss a single-field RPIM implementation and its performance in the context of signal integrity in problems that involve propagation over large distances. Our results highlight the potential of RPIM for overcoming the limitations of FDTD due to numerical phase dispersion when solving electrically large problems.

II. Approach

Time-domain Maxwell's equations for the electric and magnetic fields are combined to obtain the single-field, second order wave equation using electric field as the field variable. In the second order equation for the TM_z mode, the spatial derivatives of the electric field are replaced by their RBF approximations.

Our 2-D propagation problem is hence described by a second order equation for the z-component of the electric field:

$$\frac{\partial^2 E_z(x,y,t)}{\partial t^2} + \frac{\sigma}{\varepsilon}\frac{\partial E_z(x,y,t)}{\partial t} = \frac{1}{\mu\varepsilon}\left(\frac{\partial^2 E_z(x,y,t)}{\partial x^2} + \frac{\partial^2 E_z(x,y,t)}{\partial y^2}\right). \qquad (1)$$

In (1), ε, σ, and μ represent the electric permittivity, electric conductivity, and magnetic permeability of the media that support the wave propagation. Note that in (1), the spatial derivates have been separated from the time derivatives, as they receive a special treatment, as described thereinafter.

A detailed description of the RPIM has been submitted in [6]; nevertheless, for the reader's convenience, a few important aspects are presented in this section.

In RPIM, an arbitrary function f is approximated by an RBF expansion as

$$f(\boldsymbol{x}) \approx \sum a_j \varphi_j(\boldsymbol{x}, s). \qquad (2)$$

In (1), φ_j are RBFs, and a_j are unknown coefficients. The individual RBF basis functions in (2) are defined as

$$\varphi_j(\boldsymbol{x}, s) = \exp(-s^2\|x - x_j\|^2), \qquad (3)$$

where $\boldsymbol{x} = \{x_1, \ldots, x_M\}$ represents a set of distinct points, which can be part of a structured grid or might have no geometrical relationship to one another (e.g., a scattered point cloud). In (3), parameter s represents the shape parameter, and r is the radial distance. The unknown coefficients a_j are obtained by enforcing that the right side of (2) match f exactly at points $\{x_1, \ldots, x_M\}$, which in turn yields a system of linear equations. The second order spatial derivatives of function f are then approximated as a linear combination of the second order derivatives of φ_j (κ stands for either of the spatial coordinates):

$$\frac{\partial^2 f(x,s)}{\partial \kappa^2} \approx \sum_{j=1}^{M} a_j \frac{\partial^2 \varphi_j(x,s)}{\partial \kappa^2}. \qquad (4)$$

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The RBF of choice for this paper is the Gaussian RBF [7]:

$$\varphi(r, s) = \exp(-s^2 r^2). \quad (5)$$

Function f is the electric field, therefore (3) – (5) are used to provide an approximation of the spatial derivatives present on the right-hand side of (1). The computation of the unknown coefficients in (2), and the approximation of the spatial derivatives are performed at each time step, leading to computation times that are typically longer than FDTD. The computation of the unknown coefficients at each iteration is the major bottleneck of the simulation.

III. RESULTS

Our test scenario consisted of a 2-D rectangular domain, 1.99-m long and 0.15-m wide. A ground plane was placed at y = 0, modeled as a PEC with several layers of dielectric above it (ε_r = 5.0, σ = 0.005 S/m), as shown in Fig. 1a. The remaining three boundaries were terminated by first order Mur absorbing boundary conditions. The spatial cell sizes were Δx = 0.01 m and Δy = 0.005 m, respectively. The excitation was a point-source with a modulated gaussian signal. Simulations were run at 1.50 GHz (free-space wavelength λ_0 = 0.199 m). Simulation results are shown in Fig. 1.

RPIM simulations were run with shape parameters that were different for the x and y directions, but were maintained constant along each of the two directions. Preliminary simulations (not shown here) have confirmed that the accuracy of the method increases as the value of the shape parameter decreases. Such observations are also reported by [2]. We have also found that as the shape parameter decreases below a certain threshold, instability occurs, most likely as a result of an increase in the conditioning number of the linear system used to calculate the unknown coefficients a_j.

While simulated electric field magnitudes compare well for both RPIM and FDTD (Fig. 1b and 1c), the electric field phase distributions (Fig. 1d to f) suggest that RPIM outperforms FDTD in regards to phase velocity. An estimation of the phase velocity was done based on the wavelength measured as the distance between successive peaks in Fig.1f. The computed phase velocities for both RPIM and FDTD were normalized to c_0, the speed of the wave in vacuum. Based on this procedure, RPIM yielded a value of 0.99c_0, while FDTD yielded 0.97c_0. We observed the same behavior for other values of the spatial steps, as well.

IV. DISCUSSION

We have investigated the performance of RPIM in a 2-D propagation problem. Additional simulation results involving three-dimensional propagation will be presented at the symposium. We have found that RPIM performs comparably well or better than FDTD, and it has the potential to minimize the phase distortion in problems involving propagation over large distances. Furthermore, RPIM has an additional parameter that can be manipulated in order to increase the approximation accuracy. Our previous work [6] has also shown that the RPIM method can be accelerated and parallelized using effective domain decomposition schemes, which opens the door to solving large-scale propagation problems.

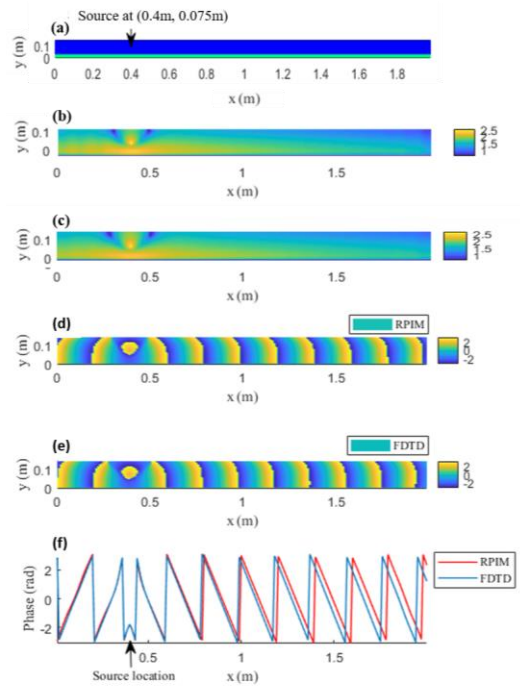

Fig. 1. Comparison between RBF time marching and FDTD for propagation over a dielectric-coated conducting ground plane. (a) The 2-D computational domain, with location of excitaton and ground plane shown in green. (b) Magnitude of electric field (decibels) calculated with RPIM; (c) Magnitude of electric field (decibels) calculated with FDTD; (d) Phase distribution (radians) calculated with RBF; (e) Phase distribution (radians) calculated with FDTD; (f) Phase variation along line $y = y_{max}/2$. Simulations were run for the same number of steps.

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